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Combining online job advertisements with probability sample data for enhanced small area estimation of job vacancies

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Auxiliary information in sample surveys

- What is the main product of National Statistical Institutes (NSIs)? *Official statistics*.
- NSIs aim for improvement:
  - → **by timeliness** – more frequent estimates,
  - → **by granularity** – more detailed level estimates.
- Typically sample designs are optimized for population-level estimates. Small domains often have:

limited or unplanned sample coverage

↓

small sample sizes

↓

high variability or unreliable estimates

- A possible solution: incorporate administrative data or other non-traditional data sources (mobile network, social media, etc.) to supplement existing probability sample data.

Job vacancy data and the problem

Data source	Target variable, y	Auxiliary data, $\mathbf{x}$
NP sample	×	✓
P sample	✓	✓

- Probability sample data on job vacancies in companies are collected in the quarterly Statistical survey on earnings.
- There is complete administrative information on the monthly number of employees, economic activity, etc.
- Transformed online job advertisement (OJA) data:
  - only partially covers the survey population;
  - as non-probability (or big data) sample is not representative;
  - roughly approximates job vacancies by nonlinear relationship.

Direct job vacancy estimation in domains

- Let $\mathcal{U}$ be the finite population and $\mathcal{U} = \mathcal{U}_1 \cup \cdots \cup \mathcal{U}_M$ be its partition into M non-overlapping domains, $|\mathcal{U}_m| = N_m$.
- The aim is to estimate domain totals

$$t_m = \sum_{i \in \mathcal{U}_m} y_i, \quad m = 1, \dots, M.$$

- The probability sample A_m is of size $n_m \leq N_m$ in the m-th domain.
- The inaccuracy of the estimator can also be expressed using the Coefficient of Variation (CV):

$$\mathrm{CV}(\hat{t}_m) = \sqrt{\mathrm{var}(\hat{t}_m)}/\hat{t}_m.$$

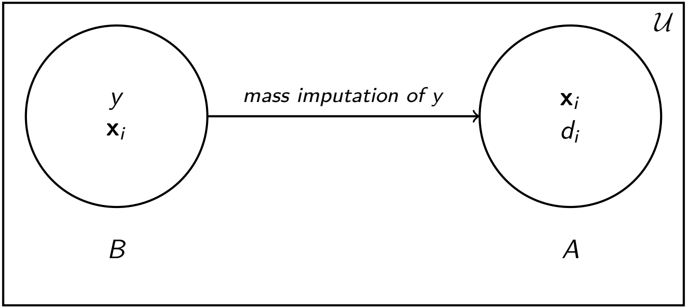
- If the sizes N_m are assumed to be known, the direct Hájek estimators of the totals t_m are

$$\hat{t}_m^{\mathrm{H}} = \frac{N_m}{\widehat{N}_m} \sum_{i \in A_m} d_i y_i \quad \text{with} \quad \widehat{N}_m = \sum_{i \in A_m} d_i, \quad m = 1, \dots, M,$$

  where $d_i = 1/\pi_i$ are design weights and π_i are the first-order inclusion probabilities.
- The variances $\psi_m^{\mathrm{H}} = \mathrm{var}(\hat{t}_m^{\mathrm{H}})$ may be too large for small n_m.

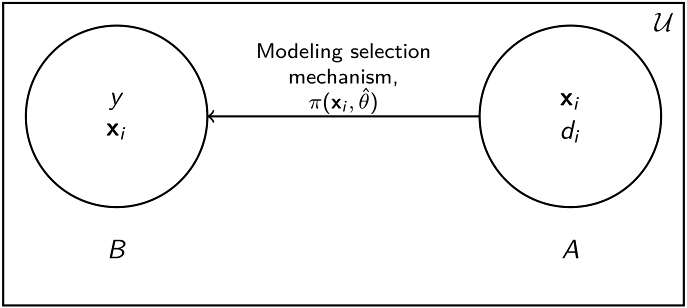
Possible cases of NP integration

- A and B – probability and non-probability samples respectively,
- y_i – the target variable for which a parameter (such as total, mean, or quantile) needs to be estimated,.
- $\mathbf{x}_i$ – auxiliary covariate vector,
- d_i – design weight of ith unit.



Possible cases of NP integration (2)

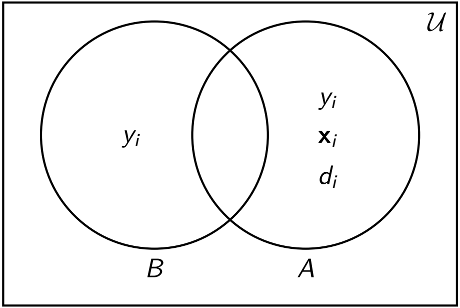
- A and B – probability and non-probability samples respectively,
- y_i – the target variable for which a parameter (such as total, mean, or quantile) needs to be estimated,.
- $\mathbf{x}_i$ – auxiliary covariate vector,
- d_i – design weight of ith unit,
- $\pi(\mathbf{x}_i, \hat{\theta}), i \in B$ – estimated propensity scores.



Possible cases of NP integration (3)

Kim & Tam (2021) regression data integration estimator:

- δ_i – inclusion into B indicator,
- w_i – calibrated weight of unit ith,
- $D(\cdot,\cdot)$ – distance function.



$$\hat{t}_m^{RegDI} = \sum_{i \in A_m} w_i \, y_i,$$

$$\mathbf{w} = \arg\min_{\mathbf{w}} D\big(\mathbf{w}, \mathbf{d}\big),$$

subject to:

$$\sum_{i \in A_m} w_i \, \delta_i = N_{B_m},$$

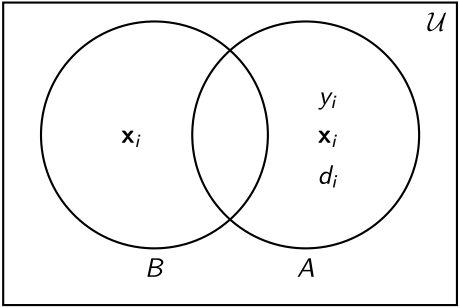
$$\sum_{i \in A_m} w_i \, \big(1 - \delta_i\big) = N_m - N_{B_m},$$

$$\sum_{i \in A_m} w_i \, \delta_i \, y_i = \sum_{i \in B_m} y_i.$$

The case of NP based on OJA

Modified regression data integration estimator based on model-calibration: *(Wu & Sitter, 2001)*

- δ_i – inclusion into B indicator,
- $D(\cdot, \cdot)$ – distance function,
- $\hat{\mu}_i = \hat{\mu}_i(\mathbf{x}_i, \hat{\theta})$ – predictions of y_i based on model that was fitted on $A \cap B$ data.



$$\hat{t}_m^{\text{MC}} = \sum_{i \in A_m} w_i \, y_i,$$

$$\mathbf{w} = \arg\min_{\mathbf{w}} D\Big(\mathbf{w}, \mathbf{d}\Big),$$

subject to:

$$\sum_{i \in A_m} w_i \, \delta_i = N_{B_m},$$

$$\sum_{i \in A_m} w_i \, \Big(1 - \delta_i\Big) = N_m - N_{B_m},$$

$$\sum_{i \in A_m} w_i \, \delta_i \, \hat{\mu}_i = \sum_{i \in B_m} \hat{\mu}_i.$$

Further small area estimation modeling

The data for the Fay–Herriot (FH) model *(Fay & Herriot, 1979)*:

- The model-calibrated estimators $\hat{t}_m^{\mathrm{MC}}$ treated as the direct estimators because they are approximately design-unbiased under certain conditions *(Wu & Sitter, 2001)*.
- Estimators $\tilde{\psi}_m^{\mathrm{MC}}$ of the variances $\psi_m^{\mathrm{MC}} = \mathrm{var}(\hat{t}_m^{\mathrm{MC}})$.
- Exactly known area-level covariates $\mathbf{z}_m = (z_{m1}, \ldots, z_{mq})'$, $q \le p$, selected from aggregates of auxiliary data $\mathbf{x}_i$, $i \in \mathcal{U}_m$.

The standard FH model is the linear mixed model

$$\hat{t}_m^{\mathrm{MC}} = \mathbf{z}'_m \beta + v_m + \varepsilon_m, \quad m = 1, \ldots, M,$$

where $\varepsilon_m \overset{\mathrm{ind}}{\sim} \mathcal{N}(0, \psi_m^{\mathrm{MC}})$ are sampling errors, $v_m \overset{\mathrm{iid}}{\sim} \mathcal{N}(0, \sigma_v^2)$ are random area effects independent of ε_m, and β are fixed effects.

EBLUP based on the FH model

The empirical best linear unbiased predictions (EBLUPs) of the domain totals t_m, $m = 1, \ldots, M$, are expressed as the linear combinations *(Fay & Herriot, 1979)*

$$\hat{t}_m^{\mathrm{FH}} = \hat{\gamma}_m \hat{t}_m^{\mathrm{MC}} + (1 - \hat{\gamma}_m)\mathbf{z}_m' \hat{\boldsymbol{\beta}} \quad \text{with} \quad \hat{\gamma}_m = \frac{\hat{\sigma}_v^2}{\tilde{\psi}_m^{\mathrm{MC}} + \hat{\sigma}_v^2},$$

and

$$\hat{\boldsymbol{\beta}} = \left(\sum_{m=1}^{M} \frac{\mathbf{z}_m \mathbf{z}_m'}{\tilde{\psi}_m^{\mathrm{MC}} + \hat{\sigma}_v^2} \right)^{-1} \sum_{m=1}^{M} \frac{\mathbf{z}_m \hat{t}_m^{\mathrm{MC}}}{\tilde{\psi}_m^{\mathrm{MC}} + \hat{\sigma}_v^2},$$

where $\hat{\sigma}_v^2$ is an estimator of the variance σ_v^2 of random area effects.

For data like job vacancies, the standard FH model should be applied to the log-transformed estimators *(Rao & Molina, 2015)*

$$\log(\hat{t}_m^{\mathrm{MC}}) \quad \text{with} \quad \mathrm{var}\left(\log(\hat{t}_m^{\mathrm{MC}})\right) \approx (\hat{t}_m^{\mathrm{MC}})^{-2}\, \mathrm{var}(\hat{t}_m^{\mathrm{MC}}).$$

Effectiveness for a single quarter

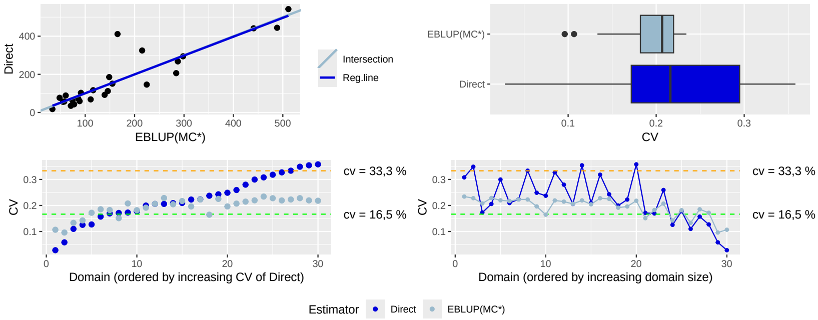


Figure 1: Comparison of direct estimates and EBLUPs for a period of 2024 Q2.
Note: good (CV $\leqslant$ 16.5%), sufficient (16.5% $<$ CV $\leqslant$ 33.3%), unreliable (CV $>$ 33.3%)

Effectiveness for multiple quarters

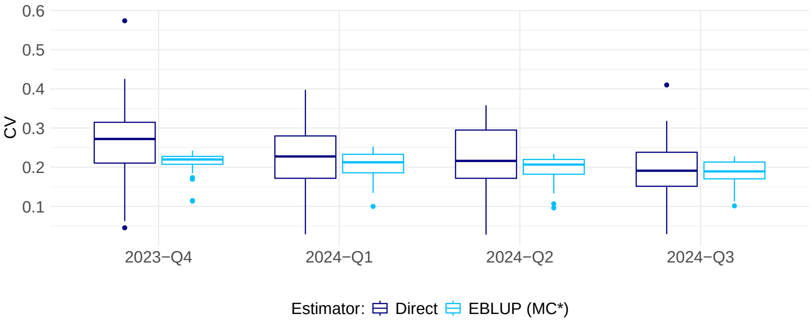


Figure 2: Comparison of direct estimates and EBLUPs.

Effectiveness for multiple quarters (2)

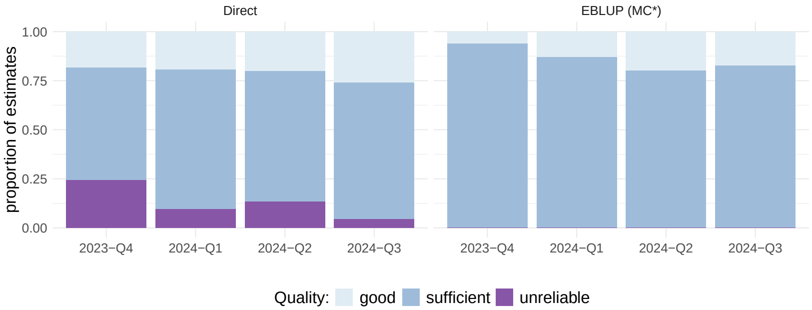


Figure 3: Trends in Direct and EBLUP estimates by quality groups.

Note: good (CV $\leqslant$ 16.5%), sufficient (16.5% $<$ CV $\leqslant$ 33.3%), unreliable (CV $>$ 33.3%).

Tools overview

- Initial preprocessing and record linkage:
  - Performed in Python using Spark for efficient data processing.
- Model building and model calibration estimates:
  - Conducted in R using the `StatMatch` and `survey` packages.
- Final EBLUP estimates and diagnostics:
  - Generated using the `emdi` package in R for small area estimation.

Literature sources

Fay, R.E., Herriot, R.A. (1979). Estimates of income for small places: an application of James-Stein procedures to census data. *J. Amer. Statist. Assoc.* 74:269–277.

Kim, J.-K., Tam, S.-M. (2021). Data integration by combining big data and survey sample data for finite population inference. *Int. Stat. Rev.* 89:382–401.

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